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Introduction

Private assets such as private equity and venture capital have long been a thorn in the side of asset allocators and chief investment officers. Their lack of liquidity makes it hard to model their return streams as an input to an asset allocation model, risk budget, or optimization function. Typically, asset allocators make assumptions about volatility and correlations based more on intuition and expectation than empirical fact.

Illiquid assets trade infrequently. And their lack of movement with the liquid financial markets increases both their liquidity risk as well as the risk that their distributional properties will not be sufficiently accounted for in the asset allocation process. The volatility and correlations with liquid asset classes as well as the skew and kurtosis all tend to be underestimated. This is part of "model risk"—that the parameters used in the asset allocation model might be misspecified. This can lead to consultants, allocators, and chief investment officers recommending a greater allocation to private assets than may be warranted by the underlying economics.

In this paper, we attempt to correct for the misspecification of illiquid assets. More specifically, we show that estimates of the parameters of the underlying distributions are routinely underestimated. This underestimation of the true risk and return factors can lead to an allocation to private assets that is not consistent with the fundamental economic value of illiquid assets. We present a new method to unsmooth illiquid asset class returns to reveal the true distributional parameters. We then recalibrate our asset allocation process with the new parameters to determine whether private assets should be increased or decreased in a diversified portfolio.

The Problem, Part 1

Burton Malkiel has explained that efficient markets make it impossible to consistently use today's stock price to predict what will happen tomorrow (Malkiel 2012). Nor can yesterday's stock price predict today's stock price. Public stock prices follow a "random walk," where the path of the stock price for a company cannot be predicted based on looking back in time. Because the acquisition of information in public markets is essentially costless and new information becomes embedded in share prices immediately, the only way to extract value in public markets is through superior fundamental analysis that leads to consistently active returns that result in positive alpha.

Such an assumption, however, may not be true for illiquid asset classes such as private equity and venture capital. In private equity, information is not freely available and the acquisition of information is costly, thus making the private equity and venture capital markets less efficient than public securities markets. Consequently, private equity and venture capital valuations are less likely to follow a random walk.

The informational asymmetries found in private equity occur for several reasons and have implications for performance measurement. First, private equity managers have significant discretion in marking to market their portfolios. Second, the value of private equity investments may not be easy to calculate. Third, private equity values often are based upon the addition of new investor capital into the portfolio company—and remain at cost until the next round of financing.

All of these factors can lead to nonsynchronous price changes between the value of private equity portfolios and the value of the securities markets. Sometimes this is called "stale pricing" and refers to the fact that private equity managers may be slow to mark up or down the value of their portfolio companies (Emery 2002). This means that private equity values may lag behind the public markets until all of the systematic risk of the market can wash through the private equity portfolio.

The Solution, Part 1

The solution for dealing with private assets with non-synchronous pricing is to use lagged betas to determine the true amount of systematic market risk embedded in the returns. In addition, once the full amount of market risk is accounted for, the investor also can determine the true amount of alpha produced by the private asset manager. This method has been documented by Anson (2002, 2007, 2013), Anson et al. (2007), Woodward (2012), Jian Fan et al. (2013), and Getmansky et al. (2004).

The idea is to expand the traditional capital asset pricing model (CAPM) to include more periods than just the current stock market return. So, for example, the returns to venture capital portfolios are regressed against the current public stock market return—consistent with CAPM—as well as the stock market returns from prior periods. These are the "lagged betas"—measuring how much of the stock market return from prior periods impacts the current returns to venture capital. How many prior periods must be included in the lagged beta equation depends upon the illiquid asset. For example, for real estate returns, Anson (2010) finds that the betas associated with the five prior quarters are statistically significant in explaining the amount of systematic market risk embedded in real estate returns.

The purpose of these prior studies was to determine the full amount of market or beta risk embedded in illiquid assets, which previously had not been fully measured. Second, these studies also served to reveal the true amount of alpha derived from illiquid assets. Using the lagged beta model, the full amount of systematic market risk now can be accounted for, which in turn reveals the remaining alpha after all of the lagged beta returns have been accounted for. In each of the papers cited above, the amount of alpha declines significantly when lagged betas are included in the CAPM factor model.

Lagged beta models fall into the general class of distributed lag models. Distributed lag models measure the impact of an independent variable X on the dependent variable Y over time. In other words, the effect of X on Y does not happen all at once but rather over a period of time.

There are two problems with distributed lag models. The first is multi-collinearity. High levels of correlation among the lagged X variables can lead to multi-collinearity which, in turn, can lead to unreliable coefficient estimates with high standard errors. In this paper, the independent variables are the quarterly returns to the public stock market. We test whether there is any correlation across public stock market returns on a quarter-by-quarter basis and we do not find any evidence of statistically significant correlation.

A second issue of distributed lag models is that they can be problematic to estimate if the lag is very long or infinite. Fortunately, we find the lag period to be only three quarters for private equity and five quarters for venture capital.

For this paper we focus on two illiquid asset classes—private equity (leveraged buyouts) and venture capital. We use data from Cambridge Associates, which, in turn, collects data about private equity and venture capital returns from foundations, pension funds, and endowments that are active investors in these two illiquid asset classes. We use the lagged beta technology cited above.[1]

Table 1 shows our results for private equity. Table 1A shows the results for a traditional CAPM single-period model to determine the amount of systematic risk associated with private equity. For the single-period model, the beta of private equity is 0.46, with a t-statistic of 9.45, and an R-squared measure of 0.44. Also, the alpha is statistically significant at 2.2 percent.[2]

When we examine table 1B, we see that the lagged betas are statistically significant at the 1-percent and 5-percent level over three prior quarters of market returns. Also, the

Table 1: Private Equity on Russell 1000, 2001–2014

A. Single-Period Analysis				
R-Squared	0.44			
Correlation Coefficient	0.66			
	Coefficients	**Standard Error**	***t*-Statistic**	***P*-value**
Intercept	0.022	0.00	5.20	0.00
RU1000	0.46	0.05	9.45	0.00
B. Multi-Period Analysis				
R-Squared	0.74			
Correlation Coefficient	0.86			
	Coefficients	**Standard Error**	***t*-Statistic**	***P*-value**
Intercept	0.016	0.00	4.17	0.00
RU1000	0.46	0.04	10.79	0.00
RU1000-1	0.09	0.04	2.25	0.03
RU1000-2	0.10	0.04	2.48	0.02
RU1000-3	0.11	0.04	2.66	0.01
Total Beta	**0.77**			

Table 2: Venture Capital on Russell 1000, 2001–2014

A. Single-Period Analysis				
R-Squared	0.32			
Correlation	0.56			
	Coefficients	**Standard Error**	***t*-Statistic**	***P*-value**
Intercept	0.008	0.004	2.00	0.05
RU1000	0.39	0.08	4.99	0.00
B. Multi-Period Analysis				
R-Squared	0.65			
Correlation	0.80			
	Coefficients	**Standard Error**	***t*-Statistic**	***P*-value**
Intercept	–0.01	0.01	–1.46	0.15
RU1000	0.37	0.055	6.24	0.00
RU1000-1	0.20	0.06	3.35	0.00
RU1000-2	0.15	0.06	2.61	0.01
RU1000-3	0.21	0.06	3.50	0.00
RU1000-4	0.10	0.06	1.75	0.09
RU1000-5	0.16	0.05	2.54	0.01
Total Beta	**1.19**			

total beta of private equity increases to 0.77 and R-squared increases to 0.74. Last, the alpha declines to 1.6 percent—an indication that a portion of what was previously thought to be alpha associated with private equity returns was nothing more than lagged or delayed systematic market return—good old-fashioned beta.

We find similar results in table 2 with respect to venture capital. In the single-period model, the beta for venture capital is only 0.39 but in the multi-period lagged model, the beta expands to 1.19—much more consistent with what we would expect for investing in more risky start-up ventures. Notice how far back the lagging occurs for venture capital: We find that the market returns for up to five previous quarters are statistically significant to measure the true amount of systematic risk embedded in venture capital returns. Also, the R-squared measure increases from 0.32 to 0.65. Last, the alpha declines from +0.8 percent in the single-period model to −1.0 percent in the multi-period model. In fact, it seems that when the full impact of systematic market risk is factored into the returns to venture capital, the alpha not only fully erodes but turns negative.[3]

Prior papers were primarily concerned with measuring the beta and alpha associated with illiquid asset classes—an important attribute when assessing asset manager performance. A remaining question to address is how does the lagged beta analysis impact the portfolio construction process? In other words, can we use the lagged beta analysis to develop a better asset allocation model with respect to including illiquid assets?

The Problem, Part 2

Even after we identify the true beta and alpha associated with an illiquid asset class, we are still left with the issue of how to blend that illiquid asset into an asset allocation model. Asset allocation models are dependent upon volatility and correlations more than they are on beta and alpha. However, illiquid assets are prone to a smoothing effect that can artificially reduce volatility and correlation estimates.

Smoothing arises because illiquid assets trade infrequently. This leads to a lag effect that reduces volatility and correlations with other asset classes. Using the raw returns for any illiquid asset class will underestimate the risk of that asset class and overstate its benefit, resulting in higher allocations to illiquid assets than would otherwise be warranted.

Much of the earlier research on how to blend illiquid assets into the asset allocation process has focused on real estate (Marcato and Key 2007; Edelstein and Quan 2006; Geltner et al. 2003). These studies used an autoregressive model to unsmooth the returns associated with raw real estate in an attempt to reveal true volatility and correlation estimates. The general model for a first order autoregressive (AR) filter is:

$$\text{Unsmoothed Return}_T = (\text{Observed Return}_T - \alpha \times \text{Observed Return}_{T-1}) / (1 - \alpha)$$

Most studies of alternative assets use a one-period model.[4] However, the model can be expanded; for example, a two-period filter model:

$$\text{Unsmoothed Return}_T = (\text{Observed Return} - \alpha_1 \times \text{Observed Return}_{T-1} - \alpha_2 \times \text{Observed Return}_{T-2}) / (1 - \alpha_1 - \alpha_2)$$

Two questions remain: (1) How many prior periods of returns to include in the autoregressive model and (2) What should be the values of the α coefficients in the model? Prior research generally has used a one- or two-period AR filter (e.g., Kinlaw et al. 2015). In some cases the alpha coefficients are chosen based more on intuition and experimentation (Marcato and Key 2007), and in other cases the coefficients are estimated from the observed autocorrelations or from a linear regression on lagged values of the dependent variable (Shaman and Stine 1988).

We answer these two questions using our lagged beta models. We use the number of periods for which we find statistically significant lagged betas as the number of autoregressive periods to include in our unsmoothing model. Second, we use our lagged beta coefficients as a way to scale our α coefficients in the autoregressive filter. We compare our results to those of Marcato and Key (2007), which used a two-period AR model with autoregressive coefficients of 0.36 and 0.10 for α_1 and α_2.

The Solution, Part 2

We present the first part of our solution for unsmoothing in table 3. Here we have provided the statistics for the raw returns to private equity and venture capital compared to our unsmoothed returns using our

Table 3: Unsmoothed Returns

	Raw Returns	Unsmoothing Using Lagged Beta Model	Unsmoothing Using Marcato and Key Model
Private Equity			
Expected Return	11.40%	11.22%	11.20%
Standard Deviation	11.44%	33.53%	18.91%
Skew	–0.3	–0.12	–0.13
Kurtosis	1.62	2.4	1.28
Correlation with Russell 1000	0.66	0.86	0.80
Venture Capital			
Expected Return	12.25%	12.11%	12.07%
Standard Deviation	11.54%	30.42%	32.62%
Skew	–0.95	–0.6	–0.42
Kurtosis	1.2	2.9	0.34
Correlation with Russell 1000	0.56	0.68	0.65

lagged beta model and the model of Marcato and Key (2007).

Examining the returns to private equity first, we can see that there is a significant increase in the volatility of returns as measured by raw returns and the returns from unsmoothing using the lagged beta model. The volatility of returns increases from 11.44 percent to 33.53 percent, more than double that initially measured. Such a large increase in volatility would be expected to have a significant impact on the asset allocation process. Conversely, the expected return declines using the lagged beta model; again, this would be expected to have an impact on the asset allocation process. Using the Marcato and Key (2007) method, the volatility of returns also increases to 18.91 percent, but not nearly as much as the full unsmoothing using the lagged beta model, and the expected return declined slightly compared to the raw returns. The skew of the return distribution remains negative and consistent across all three measurements. Interestingly, the measure of kurtosis (fatness of tails) declines using the lagged beta unsmoothing method. Last, we note that the correlation of private equity with public equity market returns increases under the lagged beta method of unsmoothing.

Turning to venture capital, we find similar results. Using the lagged beta model for unsmoothing, the expected return for venture capital declines and the volatility more than doubles. Interestingly, the Marcato and Key model provides very similar volatility and expected return estimates to the more elaborate lagged beta model—potentially, simple is better with respect to venture capital. We note again that the skew remains reasonably consistent (and negative) across the three return series although the value of kurtosis declines more significantly for the Marcato and Key method. Last, the correlation coefficient between private equity and the public equity markets increases under both the lagged beta method of unsmoothing and the Marcato and Key method—producing similar results.

Table 4 provides the full correlations for both private equity and venture capital with respect to the other asset classes that we use in our allocation model: public equity, liquid credit, Treasury bonds, real estate investment trusts (REITs), and hedge funds. We can see that the correlations with respect to public equities, credit, and government bonds increase significantly from the raw returns to the unsmoothed returns. However, for hedge funds and REITs, there is surprisingly little difference. We expected the correlations for private equity and venture capital to increase across all asset classes, but there appears to be a level of consistency with respect to the correlations among illiquid assets and other alternatives such as real estate and hedge funds. We do not have an economic explanation for this result and we suspect that there might be another factor at work that we have not identified. However, these results still apply to our asset allocation model.

For our asset allocation analysis, we use the expanded utility function of Anson (2007) to build an efficient portfolio across the asset classes listed above. The advantage of this model is that it takes into account all four moments of the distribution: mean, volatility, skew, and kurtosis. Briefly, this asset allocation model rewards higher expected returns and positive skew and penalizes negative skew, larger values of volatility, and excess kurtosis (fatter tails than a normal distribution).

Table 5 presents the results of our asset allocation model using the raw returns to private equity and venture capital compared to the unsmoothed returns. We start with private equity. Using raw returns, it is no surprise that our asset allocation model directs a large allocation to private equity—almost 46 percent. The raw returns for private equity demonstrate an expected return of more than 11.4 percent with a very low volatility of 11.44 percent—a very good risk versus return trade off. As a result, our asset allocation model grabs a large amount of private equity, mostly at the expense of public equity and government bonds.

However, when we use the unsmoothed returns, the amount allocated to private equity declines significantly—from 46 percent to 26 percent. In return, there is an increased allocation to public equity, credit, and government bonds. Once we unsmooth the private equity returns, the greatest impact is revealing its true volatility. The higher risk revealed by the unsmoothing makes private equity a less attractive asset class and the resulting portfolio is much more balanced. Not surprisingly, the expected return of the portfolio declines when the full risk profile of private equity is evaluated. However, the higher returns of private equity combined with what is still a reasonably large allocation to private equity results in a high overall return of the portfolio.

Table 5 demonstrates similar results for venture capital. Using raw returns, our asset allocation model likes venture capital even more than private equity. Using the raw returns, venture capital has an even better risk versus return trade-off than private equity. Again, it is no surprise that the asset allocation model loads up on venture capital

Table 4: Correlation Coefficients

Correlations	Smoothed Private Equity	Unsmoothed Private Equity	Smoothed Venture Capital	Unsmoothed Venture Capital
Public Equity	0.66	0.86	0.56	0.80
Credit	0.36	0.63	0.14	0.36
Government Bonds	–0.39	–0.10	–0.28	0.20
REITs	0.45	0.48	0.24	0.22
Hedge Funds	0.82	0.85	0.59	0.52

Table 5: Asset Allocation

PE Smoothed	Weight	PE Unsmoothed	Weight
Private Equity	45.79%	Private Equity	26.41%
Public Equity	17.88%	Public Equity	13.55%
Credit	18.92%	Credit	25.63%
Government Bonds	0.00%	Government Bonds	15.84%
REITs	10.09%	REITs	10.91%
Hedge Funds	7.32%	Hedge Funds	7.66%
Total	100.00%	Total	100.00%
Expected Return	10.13%	Expected Return	9.29%
Volatility	12.37%	Volatility	13.87%
Skew	–0.4	Skew	–0.19
Kurtosis	1.28	Kurtosis	2.07
VC Smoothed	**Weight**	**VC Unsmoothed**	**Weight**
Venture Capital	66.83%	Venture Capital	36.16%
Public Equity	24.29%	Public Equity	18.30%
Credit	7.10%	Credit	18.16%
Government Bonds	0.00%	Government Bonds	3.66%
REITs	1.78%	REITs	9.00%
Hedge Funds	0.00%	Hedge Funds	14.73%
Total	100.00%	Total	100.00%
Expected Return	10.57%	Expected Return	9.89%
Volatility	12.13%	Volatility	14.12%
Skew	–0.8	Skew	–0.54
Kurtosis	1.38	Kurtosis	2.11

using the raw returns—two-thirds of the portfolio is allocated to venture capital. With such a large allocation to venture capital, many of the other asset classes are short-changed, resulting in a lopsided portfolio that is almost purely driven by venture capital and public equity returns. Again, we still see high portfolio returns even when the weight to venture capital declines—the higher returns to venture capital are still the biggest driver of portfolio returns.

When we use the unsmoothed returns, however, the allocation to venture capital drops dramatically—by thirty percentage points. Granted, the asset allocation model still likes venture capital because of its high return, but now its true risk profile is taken into account in the asset allocation model. Also, the revised correlations with the other asset classes impact the amount allocated to venture capital. Last, using the unsmoothed returns, we see that there is an increase in the allocations to other asset classes. Again, the result is a much better balanced portfolio.

Conclusion

Private equity and venture capital are well-known to be illiquid asset classes. This lack of liquidity has two important implications for portfolio management. First, the illiquid nature of private assets makes it more difficult to measure the true amount of systematic market risk (beta) embedded in the return stream. Not only is the amount of beta hidden by their illiquid nature, the amount of alpha is exaggerated. Using lagged betas allows us to measure the true amount of systematic risk associated with private equity and venture capital as well as determining the true amount of alpha produced by these private asset classes.

Second, private equity and venture capital managers have significant discretion as to when they mark up or down their private assets. This can lead to a smoothing effect that hides the true volatility associated with these asset classes as well as the correlation estimates with other asset classes. Using our lagged beta model, we unsmoothed the returns to private equity and venture capital and discovered volatility estimates that were twice that derived from the raw returns. In addition, correlation estimates increased, especially with respect to public equity, credit, and government bonds.

This research helps to expand the discussion on how to allocate illiquid asset classes within an asset allocation scheme. Although the expected returns of illiquid asset classes are estimated with better accuracy, it is the estimation of the risk profile of these assets that often falls short. Volatility estimates for illiquid assets often are derived with a reference to their public market equivalents. For example, one rule of thumb is to take the volatility of the public stock market and then multiply by two to get an estimate of the volatility for venture capital. Our research shows that you can determine the values of all four moments for private assets without a need to reference the public security markets. In addition, estimates of skew and excess kurtosis change significantly when illiquid asset class returns are unsmoothed.

A key issue when unsmoothing the returns to illiquid asset classes is how many prior periods, or lags, to include. We provide a simple and intuitive way to determine the number of lags to include in the estimation model as well as a way to determine the coefficients associated with the lagged model.

We found that using the unsmoothed returns to private equity and venture capital led to very large differences in our asset allocation model compared with using the raw returns. Specifically, the unsmoothed returns resulted in a twenty-percentage-point decrease in the allocation to private equity and a 30-percent reduction in the amount allocated to venture capital. Both private equity and venture capital remain attractive asset classes with a place in portfolio construction, but their luster is less attractive under the harsh reality of their true volatility.

A potential criticism of the model is that it doesn't matter whether private equity or venture capital is efficiently priced because it cannot be traded. There are a couple of points to rebut this comment. First, private equity and venture capital are significant asset classes in most endowment, foundation, and pension plan portfolios. Regardless of whether these assets can be publicly traded, they are part of the overall asset allocation of most institutional investors. Therefore, it is important to model these asset classes as accurately as possible when constructing efficient portfolios.

Second, asset allocation models are meant to be long-term models—at least three to five years. This is sufficient time to rebalance a private equity or venture capital portfolio. Indeed, many institutional investors make use of the secondary market in private equity/venture capital to rebalance their portfolios to provide a more-efficient asset mix.

Last, the model presented relies on using lagged market variables. An argument can be made with respect to venture capital that the start-up companies financed by venture capital might be leading indicators of future stock market returns. We tested this possibility by regressing the returns to venture capital on the Russell 2000 Growth stock index—an index that represents small, emerging growth companies that are the future companies of the new economy. We found the same lagged effect, although the number of statistically significant lagged periods was four using the Russell 2000 Growth compared with the five-period lag that we report in table 2.

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Endnotes

1. The basic model for measuring lagged betas is:
   - $[R_{i,t}(PE) - \text{Tbill}] - \beta_0[R_{M,t} - \text{Tbill}] - \beta_1[R_{M,t-1} - \text{Tbill}] - \beta_2[R_{M,t-2} - \text{Tbill}]\ldots - \beta_n[R_{M,t-n} - \text{Tbill} = \beta + \beta_{i,t}$
   - We regress the returns to private equity on the current market return plus several quarters of prior market returns.
     - Betas are linearly additive, so we can take the sum of the betas to determine the true amount of systematic risk embedded in real estate portfolios.
     - $\beta_0 + \beta_1 + \beta_2 + \ldots \beta_n$ should provide a more accurate picture of how the returns to real estate co-vary with the public securities markets.

   See Anson (2002, 2007, 2013).
2. We use quarterly data, so a 2.2-percent alpha per quarter is approximately 8.8 percent per year—not a bad excess return. Similarly, the total decline in alpha when we use the multi-period model for private equity is 2.0 percent per year.
3. An interesting application of the lagged beta is whether there should be a beta adjustment when using the public market equivalent method to measure the performance of private equity. See Sorensen and Jagannathan (2015).
4. Note that the returns are not supposed to change through unsmoothing; rather unsmoothing moves the total unchanged return across the observation periods. That can result in a more accurate estimate of volatility, skew, and kurtosis.

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